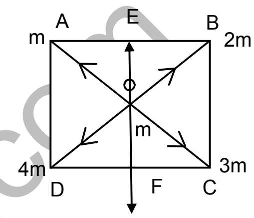
SOLUTIONS TO CONCEPTS
CHAPTER 11

1. Gravitational force of attraction,

$$F = \frac{GMm}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 10 \times 10}{(0.1)^2} = 6.67 \times 10^{-7}\ \text{N}$$

2. To calculate the gravitational force on 'm' at unline due to other mouse.



$$\overrightarrow{F_{OD}} = \frac{G \times m \times 4m}{(a/r^2)^2} = \frac{8Gm^2}{a^2}$$

$$\overrightarrow{F_{OI}} = \frac{G \times m \times 2m}{(a/r^2)^2} = \frac{6Gm^2}{a^2}$$

$$\overrightarrow{F_{OB}} = \frac{G \times m \times 2m}{(a/r^2)^2} = \frac{4Gm^2}{a^2}$$

$$\overrightarrow{F_{OA}} = \frac{G \times m \times m}{(a/r^2)^2} = \frac{2Gm^2}{a^2}$$

Resultant $\overrightarrow{F_{OF}} = \sqrt{64\left(\frac{Gm^2}{a^2}\right)^2 + 36\left(\frac{Gm^2}{a^2}\right)^2} = 10\frac{Gm^2}{a^2}$

Resultant $\overrightarrow{F_{OE}} = \sqrt{64\left(\frac{Gm^2}{a^2}\right)^2 + 4\left(\frac{Gm^2}{a^2}\right)^2} = 2\sqrt{5}\frac{Gm^2}{a^2}$

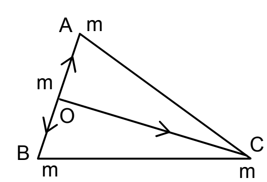
The net resultant force will be,

$$F = \sqrt{100\left(\frac{Gm^2}{a^2}\right)^2 + 20\left(\frac{Gm^2}{a^2}\right)^2 - 2\left(\frac{Gm^2}{a^2}\right) \times 20\sqrt{5}}$$

$$= \sqrt{\left(\frac{Gm^2}{a^2}\right)^2\left(120 - 40\sqrt{5}\right)} = \sqrt{\left(\frac{Gm^2}{a^2}\right)^2 (120 - 89.6)}$$

$$= \frac{Gm^2}{a^2}\sqrt{40.4} = 4\sqrt{2}\frac{Gm^2}{a^2}$$

3. a) if 'm' is placed at mid point of a side



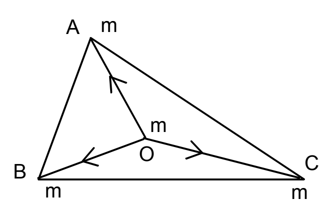
then $\overrightarrow{F_{OA}} = \frac{4Gm^2}{a^2}$ in OA direction

$\overrightarrow{F_{OB}} = \frac{4Gm^2}{a^2}$ in OB direction

Since equal & opposite cancel each other

$\overrightarrow{F_{oc}} = \frac{Gm^2}{\left[(r^3/2)a\right]^2} = \frac{4Gm^2}{3a^2}$ in OC direction

Net gravitational force on m = $\frac{4Gm^2}{a^2}$



b) If placed at O (centroid)

the $\overrightarrow{F_{OA}} = \frac{Gm^2}{(a/r_3)} = \frac{3Gm^2}{a^2}$

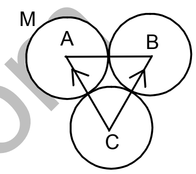
$$\overrightarrow{F_{OB}} = \frac{3Gm^2}{a^2}$$

Resultant $\vec{F} = \sqrt{2\left(\frac{3Gm^2}{a^2}\right)^2 - 2\left(\frac{3Gm^2}{a^2}\right)^2 \times \frac{1}{2}} = \frac{3Gm^2}{a^2}$

Since $\overrightarrow{F_{OC}} = \frac{3Gm^2}{a^2}$, equal & opposite to F, cancel

Net gravitational force = 0

4. $\overrightarrow{F_{CB}} = \frac{Gm^2}{4a^2}\cos 60\hat{i} - \frac{Gm^2}{4a^2}\sin 60\hat{j}$

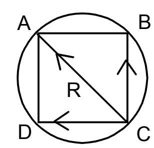


$\overrightarrow{F_{CA}} = \frac{Gm^2}{-4a^2}\cos 60\hat{i} - \frac{Gm^2}{4a^2}\sin 60\hat{j}$

$\vec{F} = \overrightarrow{F_{CB}} + \overrightarrow{F_{CA}}$

$= \frac{-2Gm^2}{4a^2}\sin 60\hat{j} = \frac{-2Gm^2}{4a^2}\frac{r_3}{2} = \frac{r_3 Gm^2}{4a^2}$

5. Force on M at C due to gravitational attraction.



$\overrightarrow{F_{CB}} = \frac{Gm^2}{2R^2}\hat{j}$

$\overrightarrow{F_{CD}} = \frac{-GM^2}{4R^2}\hat{i}$

$\overrightarrow{F_{CA}} = \frac{-GM^2}{4R^2}\cos 45\hat{j} + \frac{GM^2}{4R^2}\sin 45\hat{j}$

So, resultant force on C,

$\therefore \overrightarrow{F_C} = \overrightarrow{F_{CA}} + \overrightarrow{F_{CB}} + \overrightarrow{F_{CD}}$

$= -\frac{GM^2}{4R^2}\left(2 + \frac{1}{\sqrt{2}}\right)\hat{i} + \frac{GM^2}{4R^2}\left(2 + \frac{1}{\sqrt{2}}\right)\hat{j}$

$\therefore F_C = \frac{GM^2}{4R^2}\left(2\sqrt{2}+1\right)$

For moving along the circle, $\vec{F} = \frac{mv^2}{R}$

or $\frac{GM^2}{4R^2}\left(2\sqrt{2}+1\right) = \frac{MV^2}{R}$ or $V = \sqrt{\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)}$

6. $\frac{GM}{(R+h)^2} = \frac{6.67\times10^{-11}\times7.4\times10^{22}}{(1740+1000)^2\times10^6} = \frac{49.358\times10^{11}}{2740\times2740\times10^6}$

$= \frac{49.358\times10^{11}}{0.75\times10^{13}} = 65.8 \times 10^{-2} = 0.65\ m/s^2$

7. The linear momentum of 2 bodies is 0 initially. Since gravitational force is internal, final momentum is also zero.

So (10 kg)v_1 = (20 kg) v_2

Or $v_1 = v_2$ …(1)

Since P.E. is conserved

Initial P.E. $= \frac{-6.67\times10^{-11}\times10\times20}{1} = -13.34\times10^{-9}$ J

When separation is 0.5 m,

$-13.34 \times 10^{-9} + 0 = \dfrac{-13.34 \times 10^{-9}}{(1/2)} + (1/2) \times 10\, v_1^2 + (1/2) \times 20\, v_2^2$ …(2)

$\Rightarrow -13.34 \times 10^{-9} = -26.68 \times 10^{-9} + 5\, v_1^2 + 10\, v_2^2$

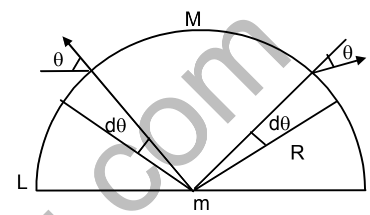
$\Rightarrow -13.34 \times 10^{-9} = -26.68 \times 10^{-9} + 30\, v_2^2$

$\Rightarrow v_2^2 = \dfrac{13.34 \times 10^{-9}}{30} = 4.44 \times 10^{-10}$

$\Rightarrow v_2 = 2.1 \times 10^{-5}$ m/s.

So, $v_1 = 4.2 \times 10^{-5}$ m/s.

8. In the semicircle, we can consider, a small element of d, then R dθ = (M/L) R dθ = dM.



$F = \dfrac{GMRd\theta m}{LR^2}$

$dF_3 = 2\,dF$ since $= \dfrac{2GMm}{LR}\sin\theta\, d\theta$.

$\therefore F = \int_0^{\pi/2} \dfrac{2GMm}{LR}\sin\theta d\theta = \dfrac{2GMm}{LR}\left[-\cos\theta\right]_0^{\pi/2}$

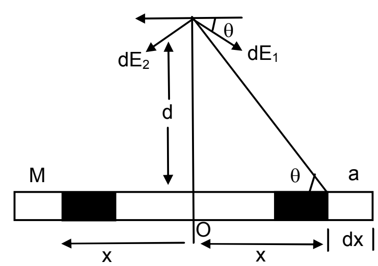
$\therefore = -2\dfrac{GMm}{LR}(-1) = \dfrac{2GMm}{LR} = \dfrac{2GMm}{L \times L/A} = \dfrac{2\pi GMm}{L^2}$

9. A small section of rod is considered at 'x' distance mass of the element = (M/L). dx = dm

$dE_1 = \dfrac{G(dm) \times 1}{(d^2 + x^2)} = dE_2$

Resultant dE = 2 dE_1 sin θ

$= 2 \times \dfrac{G(dm)}{(d^2 + x^2)} \times \dfrac{d}{\sqrt{(d^2 + x^2)}} = \dfrac{2 \times GM \times d\ dx}{L(d^2 + x^2)\left(\sqrt{(d^2 + x^2)}\right)}$



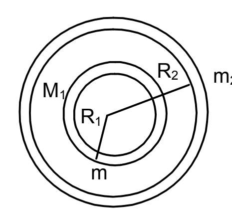
Total gravitational field

$E = \int_0^{L/2} \dfrac{2Gmd\,dx}{L(d^2 + x^2)^{3/2}}$

Integrating the above equation it can be found that,

$E = \dfrac{2GM}{d\sqrt{L^2 + 4d^2}}$

10. The gravitational force on 'm' due to the shell of M_2 is 0.



M is at a distance $\dfrac{R_1 + R_2}{2}$

Then the gravitational force due to M is given by

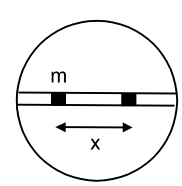
$= \dfrac{GM_1 m}{(R_1 + R_{2/2})} = \dfrac{4GM_1 m}{(R_1 + R_2)^2}$

11. Man of earth M = (4/3) $\pi R^3 \rho$

Man of the imaginary sphere, having

Radius = x, M′ = (4/3)$\pi x^3 \rho$

or $\dfrac{M'}{M} = \dfrac{x^3}{R^3}$

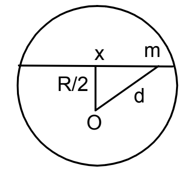


$\therefore$ Gravitational force on F = $\dfrac{GM'm}{m^2}$

or $F = \dfrac{GMx^3 m}{R^3 x^2} = \dfrac{GMmx}{R^3}$

12. Let d be the distance from centre of earth to man 'm' then

$$D = \sqrt{x^2 + \left(\frac{R^2}{4}\right)} = (1/2)\ \sqrt{4x^2 + R^2}$$

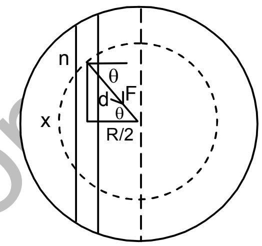


M be the mass of the earth, M′ the mass of the sphere of radius d/2.
Then M = (4/3) $\pi R^3 \rho$
M′ = (4/3)$\pi d^3\ \tau$

or $\frac{M'}{M} = \frac{d^3}{R^3}$

∴ Gravitational force is m,

$$F = \frac{Gm'm}{d^2} = \frac{Gd^3Mm}{R^3d^2} = \frac{GMmd}{R^3}$$

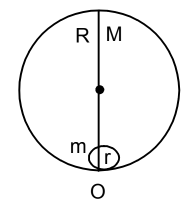


So, Normal force exerted by the wall = F cosθ.

$= \frac{GMmd}{R^3} \times \frac{R}{2d} = \frac{GMm}{2R^2}$ (therefore I think normal force does not depend on x)

13. a) m′ is placed at a distance x from 'O'.
If r < x , 2r, Let's consider a thin shell of man

$$dm = \frac{m}{(4/3)\pi r^2} \times \frac{4}{3}\pi x^3 = \frac{mx^3}{r^3}$$



Thus $\int dm = \frac{mx^3}{r^3}$

Then gravitational force F = $\frac{Gmdm}{x^2} = \frac{Gmx^3/r^3}{x^2} = \frac{Gmx}{r^3}$

b) 2r < x < 2R, then F is due to only the sphere.

$\therefore F = \frac{Gmm'}{(x-r)^2}$

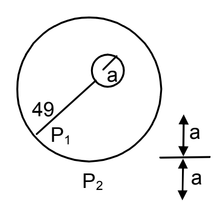
c) if x > 2R, then Gravitational force is due to both sphere & shell, then due to shell,

$F = \frac{GMm'}{(x-R)^2}$

due to the sphere = $\frac{Gmm'}{(x-r)^2}$

So, Resultant force = $\frac{Gmm'}{(x-r)^2} + \frac{GMm'}{(x-R)^2}$

14. At P_1, Gravitational field due to sphere M = $\frac{GM}{(3a+a)^2} = \frac{GM}{16a^2}$

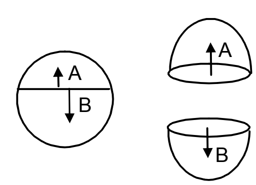


At P_2, Gravitational field is due to sphere & shell,

$= \frac{GM}{(a+4a+a)^2} + \frac{GM}{(4a+a)^2} = \frac{GM}{a^2}\left(\frac{1}{36} + \frac{1}{25}\right) = \left(\frac{61}{900}\right)\frac{GM}{a^2}$

15. We know in the thin spherical shell of uniform density has gravitational field at its internal point is zero.

At A and B point, field is equal and opposite and cancel each other so Net field is zero.



Hence, $E_A = E_B$

16. Let 0.1 kg man is x m from 2kg mass and (2 – x) m from 4 kg mass.

$$\therefore \frac{2 \times 0.1}{x^2} = -\frac{4 \times 0.1}{(2-x)^2}$$

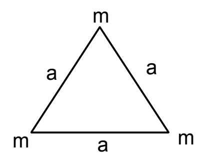
or $\frac{0.2}{x^2} = -\frac{0.4}{(2-x)^2}$

or $\frac{1}{x^2} = \frac{2}{(2-x)^2}$ or $(2 - x)^2 = 2x^2$

or $2 - x = \sqrt{2}\,x$ or $x(r_2 + 1) = 2$

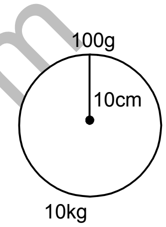
or $x = \frac{2}{2.414}$ = 0.83 m from 2kg mass.

17. Initially, the ride of Δ is a
To increase it to 2a,



work done = $\frac{Gm^2}{2a} + \frac{Gm^2}{a} = \frac{3Gm^2}{2a}$

18. Work done against gravitational force to take away the particle from sphere,



$= \frac{G \times 10 \times 0.1}{0.1 \times 0.1} = \frac{6.67 \times 10^{-11} \times 1}{1 \times 10^{-1}} = 6.67 \times 10^{-10}$ J

19. $\vec{E}$ = (5 N/kg) $\hat{i}$ + (12 N/kg) $\hat{j}$

a) $\vec{F} = \vec{E}\,m$

= 2kg [(5 N/kg) $\hat{i}$ + (12 N/kg) $\hat{j}$] = (10 N) $\hat{i}$ + (12 N) $\hat{j}$

$|\vec{F}| = \sqrt{100 + 576}$ = 26 N

b) $\vec{V} = \vec{E}\,r$

At (12 m, 0), $\vec{V}$ = – (60 J/kg) $\hat{i}$ $|\vec{V}|$ = 60 J

At (0, 5 m), $\vec{V}$ = – (60 J/kg) $\hat{j}$ $|\vec{V}|$ = – 60 J

c) $\Delta\vec{V} = \int_{(0,0)}^{(1,2,5)} \vec{E}\,mdr = \left[\left[(10N)\hat{i} + (24N)\hat{j}\right]r\right]_{(0,0)}^{(12,5)}$

= – (120 J $\hat{i}$ + 120 J $\hat{i}$) = 240 J

d) $\Delta v = -\left[r(10N\hat{i} + 24N\hat{j})\right]_{(12m,0)}^{(0,5m)}$

= –120 $\hat{j}$ + 120 $\hat{i}$ = 0

20. a) V = (20 N/kg) (x + y)

$\frac{GM}{R} = \frac{MLT^{-2}}{M}L$ or $\frac{M^{-1}L^3T^{-2}M^1}{L} = \frac{ML^2T^{-2}}{M}$

Or $M^0 L^2 T^{-2} = M^0 L^2 T^{-2}$

$\therefore$ L.H.S = R.H.S

b) $\vec{E}_{(x,y)}$ = – 20(N/kg) $\hat{i}$ – 20(N/kg) $\hat{j}$

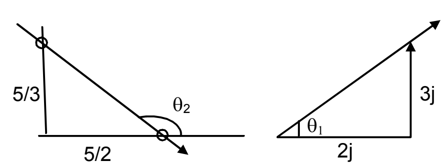
c) $\vec{F} = \vec{E}m$

= 0.5kg [– (20 N/kg) $\hat{i}$ – (20 N/kg) $\hat{j}$ = – 10N $\hat{i}$ - 10 N $\hat{j}$

$\therefore$ $|\vec{F}| = \sqrt{100 + 100} = 10\sqrt{2}$ N

21. $\vec{E} = 2\hat{i} + 3\hat{j}$

The field is represented as



$\tan\theta_1 = 3/2$
Again the line 3y + 2x = 5 can be represented as
$\tan\theta_2 = -2/3$
$m_1 m_2 = -1$
Since, the direction of field and the displacement are perpendicular, is done by the particle on the line.

22. Let the height be h

$\therefore (1/2)\ \frac{GM}{R^2} = \frac{GM}{(R+h)^2}$

Or $2R^2 = (R + h)^2$

Or $\sqrt{2}\ R = R + h$

Or $h = (r_2 - 1)R$

23. Let g′ be the acceleration due to gravity on mount everest.

$g' = g\left(1 - \frac{2h}{R}\right)$

$= 9.8\left(1 - \frac{17696}{6400000}\right) = 9.8\ (1 - 0.00276) = 9.773\ m/s^2$

24. Let g′ be the acceleration due to gravity in mine.

Then $g' = g\left(1 - \frac{d}{R}\right)$

$= 9.8\ \left(1 - \frac{640}{6400 \times 10^3}\right) = 9.8 \times 0.9999 = 9.799\ m/s^2$

25. Let g′ be the acceleration due to gravity at equation & that of pole = g

$g' = g - \omega^2 R$

$= 9.81 - (7.3 \times 10^{-5})^2 \times 6400 \times 10^3$

$= 9.81 - 0.034$

$= 9.776\ m/s^2$

$mg' = 1\ kg \times 9.776\ m/s^2$

$= 9.776\ N$ or $0.997\ kg$

The body will weigh 0.997 kg at equator.

26. At equator, $g' = g - \omega^2 R$ …(1)

Let at ‘h’ height above the south pole, the acceleration due to gravity is same.

Then, here $g' = g\ \left(1 - \frac{2h}{R}\right)$ …(2)

$\therefore\ g - \omega^2 R = g\ \left(1 - \frac{2h}{R}\right)$

or $1 - \frac{\omega^2 R}{g} = 1 - \frac{2h}{R}$

or $h = \frac{\omega^2 R^2}{2g} = \frac{(7.3 \times 10^{-5})^2 \times (6400 \times 10^3)^2}{2 \times 9.81} = 11125\ N = 10Km$ (approximately)

27. The apparent ‘g’ at equator becomes zero.

i.e. $g' = g - \omega^2 R = 0$

or $g = \omega^2 R$

or $\omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{6400 \times 10^3}} = \sqrt{1.5 \times 10^{-6}} = 1.2 \times 10^{-3}$ rad/s.

$T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{1.2 \times 10^{-3}} = 1.5 \times 10^{-6}$ sec. = 1.41 hour

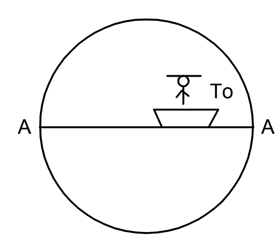
28. a) Speed of the ship due to rotation of earth $v = \omega R$

b) $T_0 = mgr = mg - m\omega^2 R$

$\therefore\ T_0 - mg = m\omega^2 R$

c) If the ship shifts at speed ‘v’

$T = mg - m\omega_1^2 R$



$= T_0 - \left(\frac{(v-\omega R)^2}{R^2}\right)R$

$= T_0 - \left(\frac{v^2+\omega^2R^2-2\omega Rv}{R}\right)$ m

$\therefore$ T = T_0 + 2ωv m

29. According to Kepler's laws of planetary motion,

$T^2 \alpha R^3$

$\frac{{T_m}^2}{{T_e}^2} = \frac{{R_{ms}}^3}{{R_{es}}^3}$

$\left(\frac{R_{ms}}{R_{es}}\right)^3 = \left(\frac{1.88}{1}\right)^2$

$\therefore \frac{R_{ms}}{R_{es}} = (1.88)^{2/3} = 1.52$

30. $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$27.3 = 2 \times 3.14\sqrt{\frac{(3.84\times10^5)^3}{6.67\times10^{-11}\times M}}$

or $2.73 \times 2.73 = \frac{2\times3.14\times(3.84\times10^5)^3}{6.67\times10^{-11}\times M}$

or $M = \frac{2\times(3.14)^2\times(3.84)^3\times10^{15}}{3.335\times10^{-11}(27.3)^2} = 6.02 \times 10^{24}$ kg

$\therefore$ mass of earth is found to be 6.02×10^{24} kg.

31. $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$\Rightarrow 27540 = 2 \times 3.14\sqrt{\frac{(9.4\times10^3\times10^3)^3}{6.67\times10^{-11}\times M}}$

or $(27540)^2 = (6.28)^2 \frac{(9.4\times10^6)^2}{6.67\times10^{-11}\times M}$

or $M = \frac{(6.28)^2\times(9.4)^3\times10^{18}}{6.67\times10^{-11}\times(27540)^2} = 6.5 \times 10^{23}$ kg.

32. a) $V = \sqrt{\frac{GM}{r+h}} = \sqrt{\frac{gr^2}{r+h}}$

$= \sqrt{\frac{9.8\times(6400\times10^3)^2}{10^6\times(6.4+2)}} = 6.9 \times 10^3$ m/s = 6.9 km/s

b) K.E. = (1/2) mv^2

= (1/2) 1000 × (47.6 × 10^6) = 2.38 × 10^{10} J

c) $P.E. = \frac{GMm}{-(R+h)}$

$= -\frac{6.67\times10^{-11}\times6\times10^{24}\times10^3}{(6400+2000)\times10^3} = -\frac{40\times10^{13}}{8400} = -4.76 \times 10^{10}$ J

d) $T = \frac{2\pi(r+h)}{V} = \frac{2\times3.14\times8400\times10^3}{6.9\times10^3} = 76.6 \times 10^2$ sec = 2.1 hour

33. Angular speed f earth & the satellite will be same

$\frac{2\pi}{T_e} = \frac{2\pi}{T_s}$

or $\frac{1}{24\times3600} = \frac{1}{2\pi\sqrt{\frac{(R+h)^3}{gR^2}}}$ or 12 I 3600 = 3.14 $\sqrt{\frac{(R+h)^3}{gR^2}}$

or $\frac{(R+h)^2}{gR^2} = \frac{(12\times3600)^2}{(3.14)^2}$ or $\frac{(6400+h)^3\times10^9}{9.8\times(6400)^2\times10^6} = \frac{(12\times3600)^2}{(3.14)^2}$

or $\frac{(6400+h)^3\times10^9}{6272\times10^9}$ = 432 × 10^4

or $(6400 + h)^3$ = 6272 × 432 × 10^4

or 6400 + h = $(6272 \times 432 \times 10^4)^{1/3}$

or h = $(6272 \times 432 \times 10^4)^{1/3} - 6400$

= 42300 cm.

b) Time taken from north pole to equator = (1/2) t

= (1/2) × 6.28 $\sqrt{\frac{(43200+6400)^3}{10\times(6400)^2\times10^6}}$ = 3.14 $\sqrt{\frac{(497)^3\times10^6}{(64)^2\times10^{11}}}$

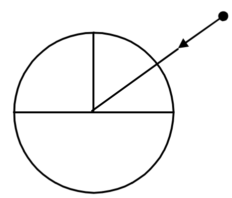
= 3.14 $\sqrt{\frac{497\times497\times497}{64\times64\times10^5}}$ = 6 hour.

34. For geo stationary satellite,

r = 4.2 × 10^4 km

h = 3.6 × 10^4 km

Given mg = 10 N



mgh = mg $\left(\frac{R^2}{(R+h)^2}\right)$

= 10 $\left[\frac{(6400\times10^3)^2}{(6400\times10^3+3600\times10^3)^2}\right] = \frac{4096}{17980}$ = 0.23 N

35. T = $2\pi\sqrt{\frac{R_2^{\,3}}{gR_1^{\,2}}}$

Or $T^2 = 4\pi^2 \frac{R_2^{\,3}}{gR_1^{\,2}}$

Or g = $\frac{4\pi^2}{T^2}\frac{R_2^{\,3}}{R_1^{\,2}}$

∴ Acceleration due to gravity of the planet is = $\frac{4\pi^2}{T^2}\frac{R_2^{\,3}}{R_1^{\,2}}$

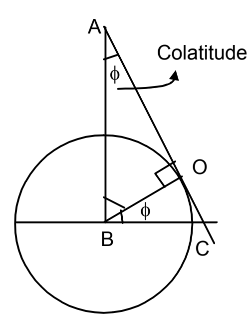
36. The colattitude is given by ϕ.

$\angle OAB = 90° - \angle ABO$

Again $\angle OBC = \phi = \angle OAB$

∴ $\sin\phi = \frac{6400}{42000} = \frac{8}{53}$

∴ $\phi = \sin^{-1}\left(\frac{8}{53}\right) = \sin^{-1} 0.15$.



37. The particle attain maximum height = 6400 km.
On earth's surface, its P.E. & K.E.

$E_e = (1/2)\ mv^2 + \left(\frac{-GMm}{R}\right)$ …(1)

In space, its P.E. & K.E.

$E_s = \left(-\frac{GMm}{R+h}\right) + 0$

$E_s = \left(-\frac{GMm}{2R}\right)$ …(2) $(\because\ h = R)$

Equating (1) & (2)

$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{2R}$

Or $(1/2)\ mv^2 = GMm\left(-\frac{1}{2R} + \frac{1}{R}\right)$

Or $v^2 = \frac{GM}{R}$

$= \frac{6.67\times10^{-11}\times6\times10^{24}}{6400\times10^3}$

$= \frac{40.02\times10^{13}}{6.4\times10^6}$

$= 6.2 \times 10^7 = 0.62 \times 10^8$

Or $v = \sqrt{0.62\times10^8} = 0.79 \times 10^4$ m/s = 7.9 km/s.

38. Initial velocity of the particle = 15km/s
Let its speed be 'v' at interstellar space.

$\therefore (1/2)\ m[(15 \times 10^3)^2 - v^2] = \int_R^\infty \frac{GMm}{x^2}dx$

$\Rightarrow (1/2)\ m[(15 \times 10^3)^2 - v^2] = GMm\left[-\frac{1}{x}\right]_R^\infty$

$\Rightarrow (1/2)\ m[(225 \times 10^6) - v^2] = \frac{GMm}{R}$

$\Rightarrow 225 \times 10^6 - v^2 = \frac{2\times6.67\times10^{-11}\times6\times10^{24}}{6400\times10^3}$

$\Rightarrow v^2 = 225 \times 10^6 - \frac{40.02}{32} \times 10^8$

$\Rightarrow v^2 = 225 \times 10^6 - 1.2 \times 10^8 = 10^8\ (1.05)$

Or $v = 1.01 \times 10^4$ m/s or
= 10 km/s

39. The man of the sphere = 6×10^{24} kg.
Escape velocity = 3×10^8 m/s

$V_c = \sqrt{\frac{2GM}{R}}$

Or $R = \frac{2GM}{{V_c}^2}$

$= \frac{2\times6.67\times10^{-11}\times6\times10^{24}}{(3\times10^8)^2} = \frac{80.02}{9} \times 10^{-3} = 8.89\times 10^{-3}$ m $\approx$ 9 mm.

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